

PROBABILITY OF FAILURE AND RISK ASSESSMENT  
OF PROPULSION STRUCTURAL COMPONENTS526-39  
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## ABSTRACT

The probabilistic structural analysis method (PSAM) has been developed to analyze the effects of fluctuating loads, variable material properties, and uncertain analytical models especially for high performance structures such as SSME turbopump blades. In the deterministic approach, uncertainties in the responses were not quantified and the actual safety margin remains unknown. Risk is calculated after expensive service experience. However, probabilistic structural analysis provides a rational alternative method to quantify uncertainties in the structural performance and the durability. NESSUS (Numerical Evaluation of Stochastic Structures Under Stress) has been developed by SwRI under the PSAM research project sponsored by NASA Lewis. It is a probabilistic structural analysis computer code which integrates finite element methods and reliability algorithms, capable of predicting the probability distributions of structural response variables such as stress, displacement, natural frequencies, and buckling loads etc. In the design and analysis practice, it is also important to assess the risk associated with a new or existing structure for safety or serviceability consideration. Probable failure criteria should be identified for different structures due to various structural functions. For example, failure events such as stress greater than strength or the displacement exceeds maximum allowable are often used for the reliability assessment. Probability of occurrence of those failure events can be determined once the probability distributions of the stress or displacement are calculated by NESSUS.

In undertaking a reliability/risk analysis, all suspected sources in uncertainties should be taken into account in order to control the probability of failure in service environments within an acceptable range. Reliability and risk obtained by a probabilistic structural analysis can be useful in evaluating the traditional design, setting quality control requirements and inspection intervals. It can also be used to identify candidate material and design concepts in the absence of technology base. In this presentation, a risk/cost assessment and a reliability analysis using NESSUS with a generic probabilistic material model is described.

The risk assessment includes the initial cost and the cost due to failure. The initial cost is defined as the cost for component service readiness which can be a function of several key design variables. The consequential cost is the cost incurred due to failure. Total cost is the sum of initial cost and a fraction of consequential cost. The fraction is weighted by the probability of failure. Since the lower initial cost is often associated with higher risk for the structural failure, higher initial cost

will normally reduce the risk. How to minimize the total cost for an acceptable structural reliability/risk is also described in this presentation.

In previous studies, probability distributions of material properties such as Young's modulus, thermal expansion coefficient, and material strength were assumed. In the present study, a generic probabilistic material model is incorporated in NESSUS. The assumption for this model is that the probability distributions of material properties are simulated by using primitive random variables. These variables can be temperature, stress, fatigue cycles, etc. The model is especially attractive for the structures under high temperature environments. A risk analysis of an SSME blade is performed under this concept. Since the material properties are functions of stress, yet stress is a function of material properties in an implicit way, an iterative procedure is necessary to obtain a convergent solution for stress and material properties. During the iteration process, the joint cumulative distribution functions of nodal stresses are required in order to apply the generic probabilistic material model. It is found that only a few iterations are needed for a converged result. From this analysis, a Risk-Fatigue cycle curve is developed for critical locations. This curve is useful for assessing the risk of existing structures. For instance, at a given acceptable risk level, the number of fatigue cycles to initial local failure can be determined. With this information available, criteria can be set for quality control, inspection intervals and retirement for cause.

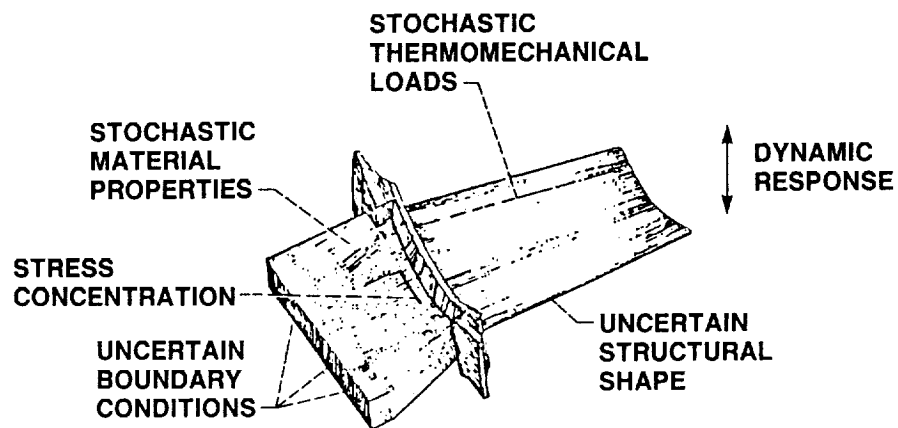
In summary, a reliability/risk cost methodology has been developed. It consists of a probabilistic structural analysis by NESSUS and a generic probabilistic material model. The methodology is versatile and particularly applicable to high temperature structures where data is difficult to obtain. It is demonstrated by using it to assess the risk associated with fatigue cycles to initiate local failure.

## OBJECTIVE

DEVELOP A METHODOLOGY TO EVALUATE PROBABILITY OF FAILURE  
AND PERFORM A RISK ASSESSMENT USING NESSUS AND A GENERIC  
PROBABILISTIC MATERIAL MODEL OF COMPLEX STRUCTURES  
OPERATING IN HIGH TEMPERATURE SERVICE ENVIRONMENTS

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## UNCERTAINTIES IN THE PROBABILISTIC STRUCTURAL ANALYSIS



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# PROBABILISTIC STRUCTURAL ANALYSIS

## BY NESSUS & A GENERIC PROBABILISTIC MATERIAL MODEL

### ITERATION 0

STEP 1: CALCULATE THE PROBABILITY DISTRIBUTIONS OF MATERIAL PROPERTIES USING THE GENERIC MATERIAL MODEL WITHOUT STRESS AND FATIGUE CYCLES

STEP 2: CALCULATE THE NODAL STRESS BY NESSUS

### ITERATION 1

STEP 1: CALCULATE THE PROBABILITY DISTRIBUTIONS OF MATERIAL PROPERTIES USING COMPLETE GENERIC MATERIAL MODEL

STEP 2: CALCULATE THE NODAL STRESS BY NESSUS WITH UPDATED MATERIAL PROPERTIES

### ITERATION 2

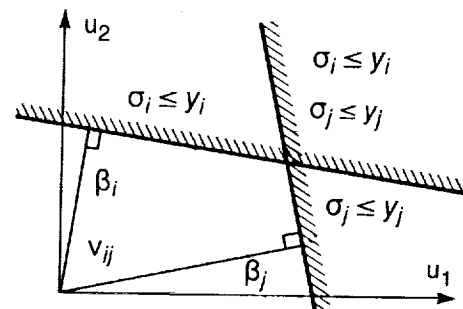
(REPEAT THE ITERATIONS UNTIL THE PROBABILITY DISTRIBUTIONS OF MATERIAL PROPERTIES AND STRESS HAVE CONVERGED)

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## JOINT CUMULATIVE DISTRIBUTION FUNCTION OF NODAL STRESSES

$\sigma_i$  &  $\sigma_j$  CALCULATED BY FIRST-ORDER SECOND-MOMENT METHOD

$\sigma_i$  = NODAL STRESS  
 $y_i$  = REAL VALUE  
 $\beta_i$  = RELIABILITY INDEX  
 $\Phi$  = NORMAL CDF  
 $\phi$  = NORMAL PDF  
 $\rho_{ij} = \cos v_{ij}$



NORMALIZED COORDINATE SYSTEM

$$P(\sigma_i \leq y_i \text{ and } \sigma_j \leq y_j) = \Phi(-\beta_i)\Phi(-\beta_j) + \int_0^{\rho_{ij}} \phi(-\beta_i - \beta_j z) dz$$

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## GENERIC PROBABILISTIC MATERIAL PROPERTY MODEL IN TERMS OF PRIMITIVE VARIABLES

$$M_P = M_{P0} \left[ \frac{T_F - T}{T_F - T_0} \right]^n \left[ \frac{S_F - \sigma}{S_F - \sigma_0} \right]^p \left[ \frac{\log N_{MF} - \log N_M}{\log N_{MF} - \log N_{M0}} \right]^q$$

### PRIMITIVE VARIABLES

$M_P$  = MATERIAL PROPERTY

$T$  = TEMPERATURE

$S$  = STRENGTH

$\sigma$  = STRESS

$N_M$  = MECHANICAL CYCLES

### SUBSCRIPTS

$F$  = FINAL CHARACTERISTIC VALUE

$0$  = REFERENCE PROPERTY

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## PRIMITIVE VARIABLE PROBABILITY DISTRIBUTIONS FOR PROBABILISTIC MATERIAL PROPERTY MODEL

VARIABLE	DISTRIBUTION TYPE	MEAN	STANDARD DEVIATION	
			(VALUE)	(% OF MEAN)
$T_F$	NORMAL	2750 °F	51.4 °F	2.0
$T_0$	NORMAL	68 °F	2.04 °F	3.0
$S_F$	NORMAL	212.0 ksi	10.6 ksi	5.0
$\sigma_0$	CONSTANT	0	0	0
$N_{MF}$	LOGNORMAL	$10^8$	$5 \times 10^6$	5.0
$N_{M0}$	LOGNORMAL	$10^3$	50	5.0
$n$	NORMAL	0.25		3.0
$p$	NORMAL	0.25		3.0
$q$	NORMAL	0.25		3.0

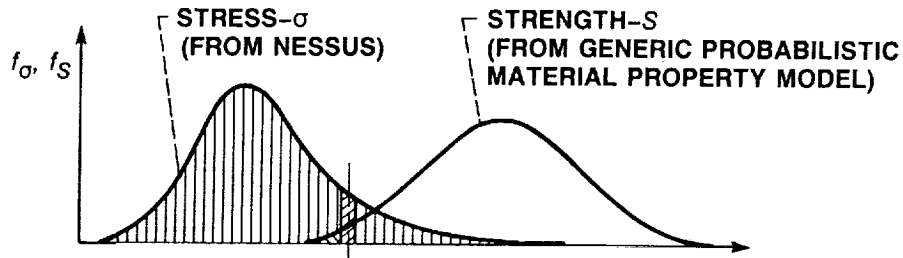
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## RISK ASSESSMENT

$$P_f = P(\sigma \geq S)$$

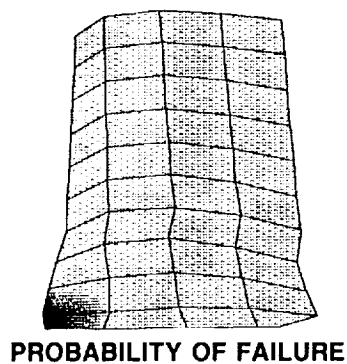
PROBABILITY OF FAILURE
STRESS
STRENGTH

$$P_f = \int_{-\infty}^{\infty} \left( \int_{-\infty}^x f_S(s) ds \right) f_{\sigma}(x) dx$$



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### PROBABILITY OF LOCAL FAILURE CAN BE QUANTIFIED (STRESS EXCEEDS STRENGTH)



PROBABILITY OF FAILURE

.0869  
.0790  
.0711  
.0632  
.0553  
.0474  
.0395  
.0316  
.0237  
.0158  
.0079  
0

PROBABILITY

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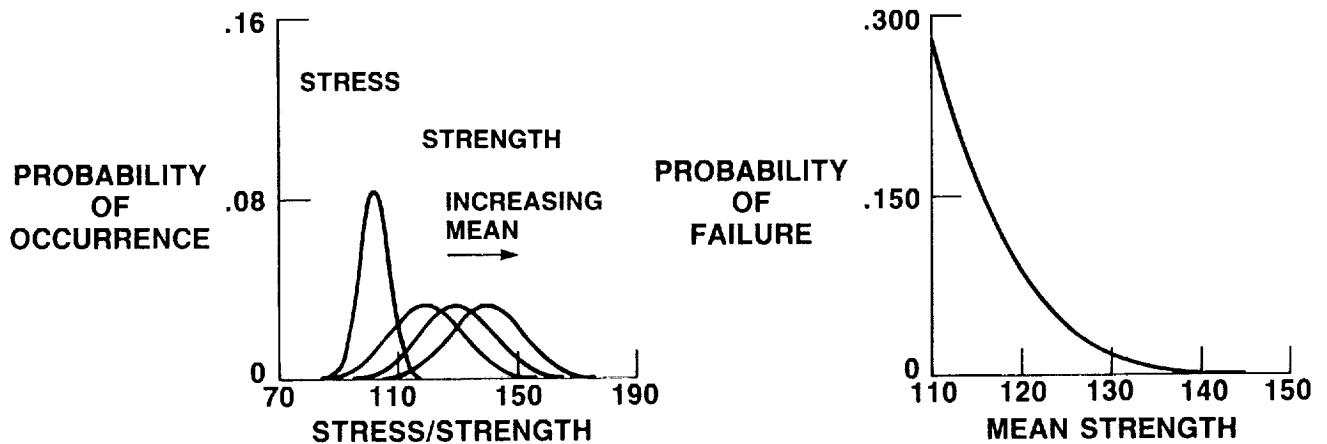
## HYPOTHESIS FOR QUANTIFYING ACCEPTANCE OF RISK

$C_i$	INITIAL COST	=	COST OF COMPONENT SERVICE READINESS
$C_f$	CONSEQUENTIAL COST	=	COST DUE TO FAILURE OCCURRENCE
$P_f$		=	PROBABILITY OF FAILURE
$C_t$		=	TOTAL COST

$$C_t = C_i + P_f C_f$$

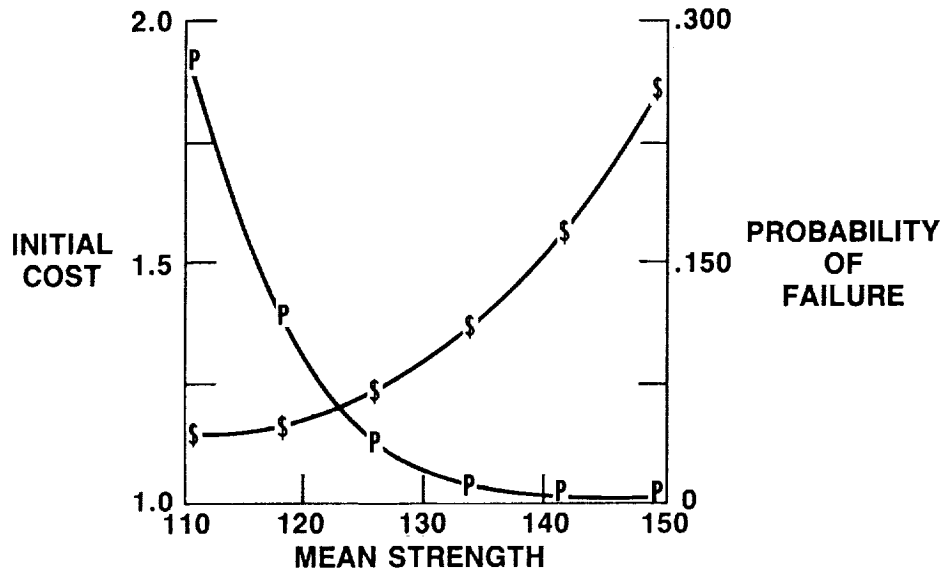
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## PROBABILITY OF FAILURE CAN BE QUANTIFIED IN TERMS OF IMPROVED MEAN STRENGTH



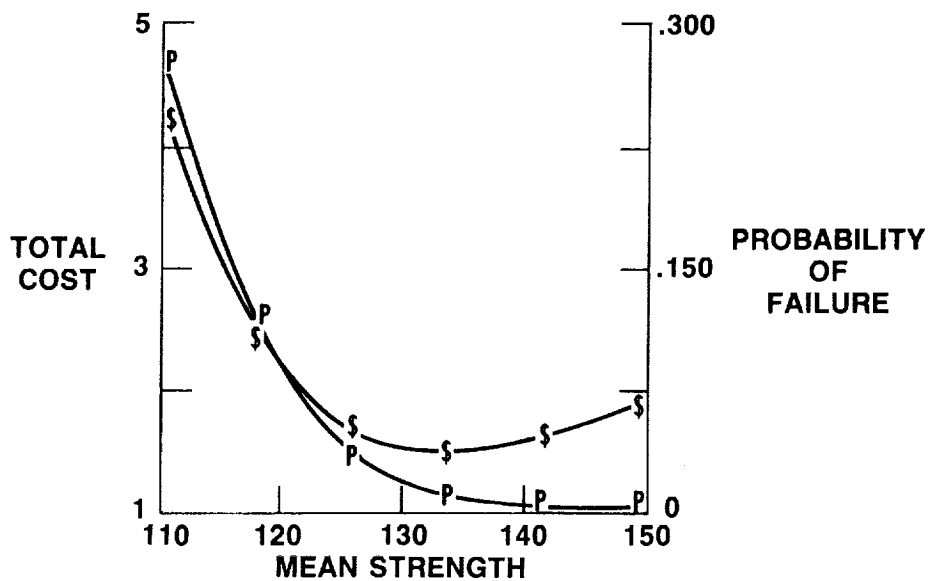
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**THE INITIAL COST TO IMPROVE THE STRUCTURAL RELIABILITY CAN BE QUANTIFIED IN TERMS OF MEAN STRENGTH (GIVEN QUALITY)**



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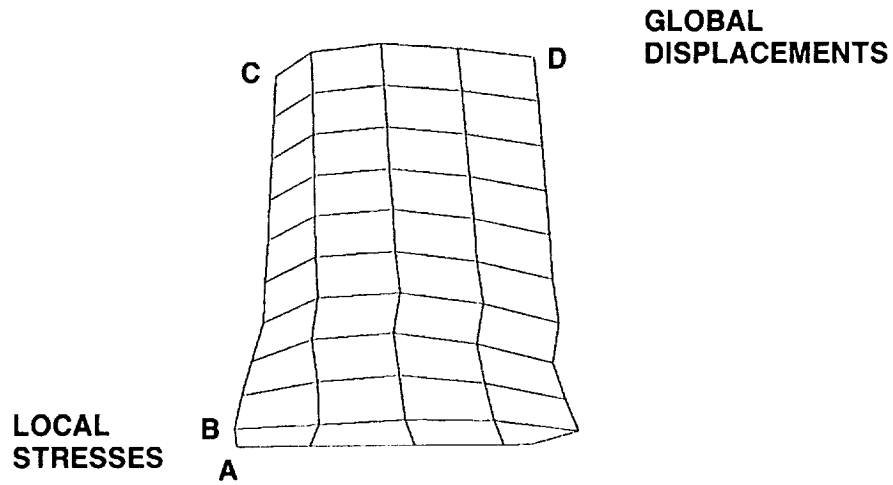
**THE TOTAL COST TO IMPROVE THE STRUCTURAL RELIABILITY CAN BE QUANTIFIED IN TERMS OF MEAN STRENGTH (GIVEN QUALITY)**



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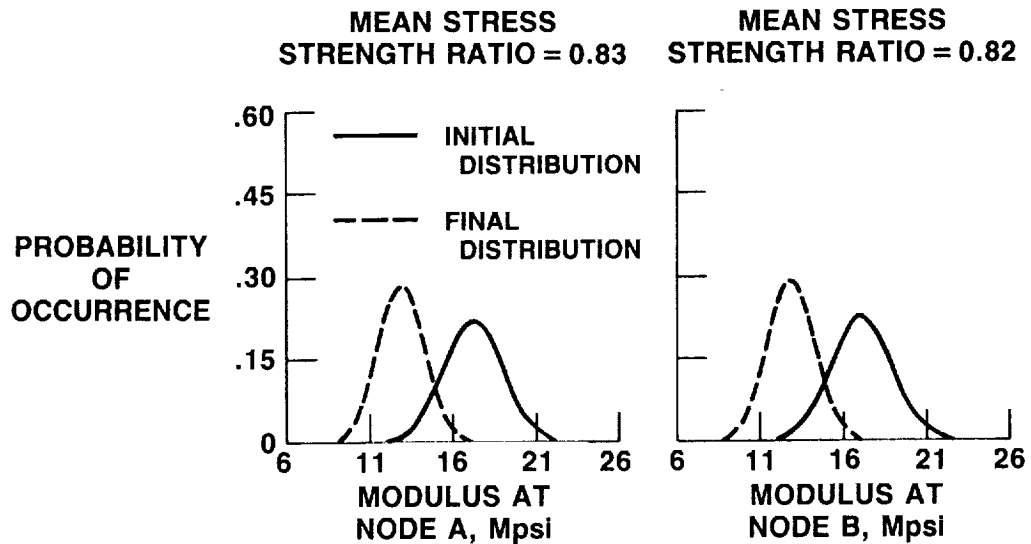


# SSME BLADE SHOWING LOCATIONS WHERE PROBABILISTIC STRUCTURAL RESPONSE WAS EVALUATED



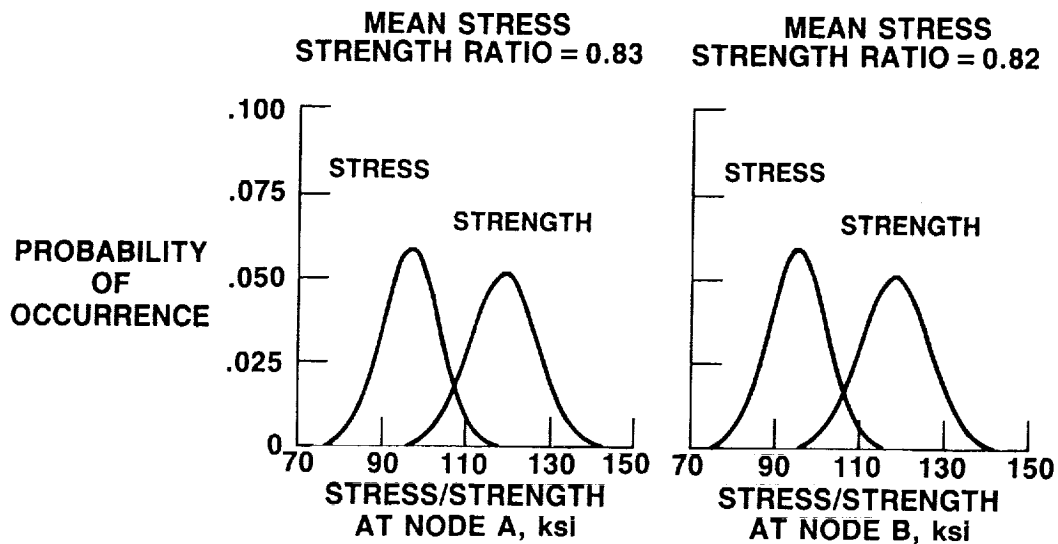
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## PROBABILISTIC MODULUS SIMULATED BY USING THE GENERIC PROBABILISTIC MATERIAL PROPERTY MODEL



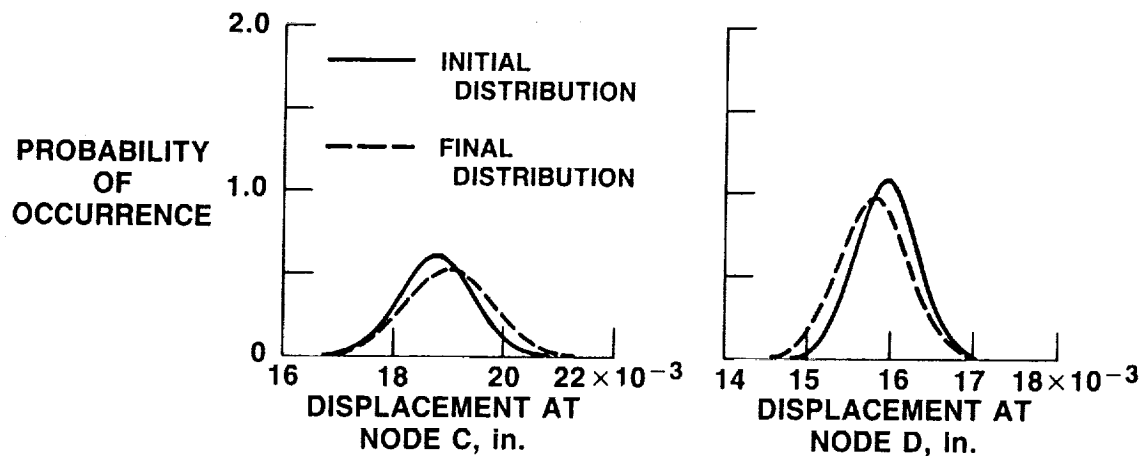
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# **PROBABILISTIC FATIGUE STRESS/STRENGTH SIMULATED BY USING THE GENERIC PROBABILISTIC MATERIAL PROPERTY MODEL**



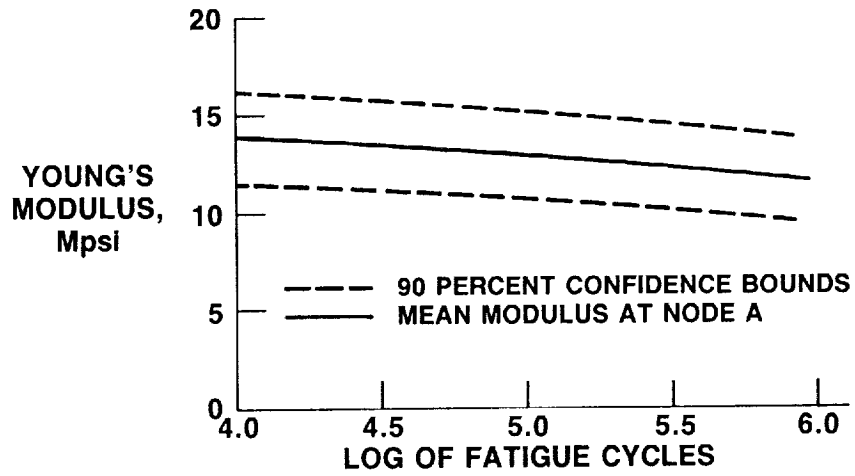
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## **PROBABILISTIC DISPLACEMENTS CALCULATED BY NESSUS**



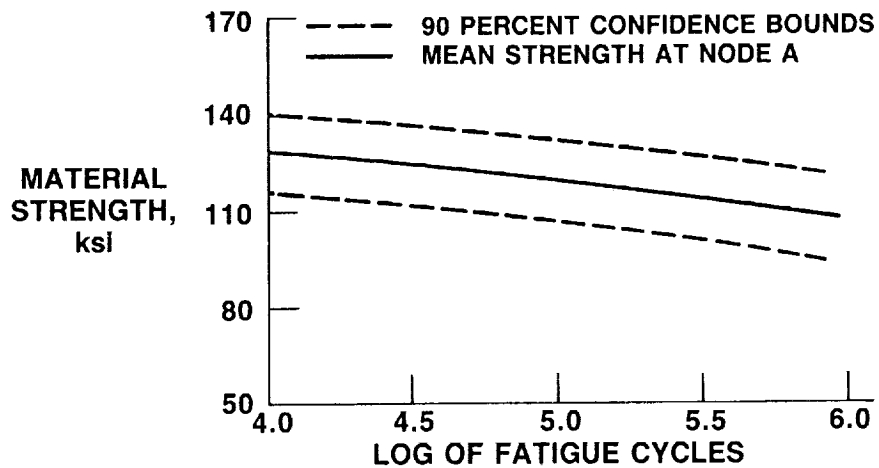
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## PROBABILISTIC CYCLIC LOAD EFFECTS ON MODULUS



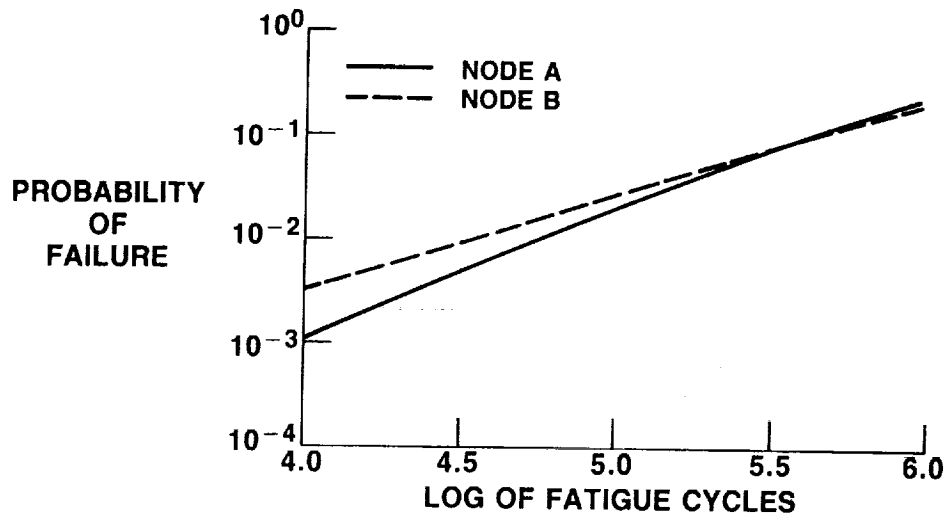
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## PROBABILISTIC CYCLIC LOAD EFFECTS ON STRENGTH



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## PROBABILITY OF LOCAL FAILURE DUE TO FATIGUE CYCLES



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## CONCLUDING REMARKS

- A METHODOLOGY HAS BEEN DEVELOPED FOR PERFORMING RELIABILITY/RISK COST BASED ASSESSMENTS OF AEROSPACE PROPULSION STRUCTURES
- THE METHODOLOGY CONSISTS OF PROBABILISTIC STRUCTURAL ANALYSIS (NESSUS) AND A PROBABILISTIC GENERIC MATERIAL PROPERTY MODEL
- THE METHODOLOGY IS ILLUSTRATED BY USING IT TO ASSESS THE RISK FOR QUALITY CONTROL, STRENGTH IMPROVEMENT, AND FATIGUE CYCLES TO INITIATE LOCAL FAILURE

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